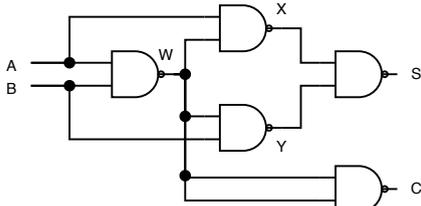


# A Level CS C19 Logic Circuits and Boolean algebra

**The half adder:** a circuit which performs binary addition of two individual bits

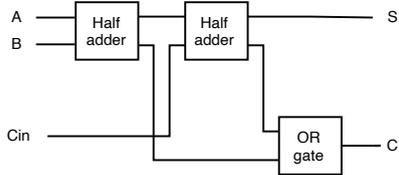
**sum bit(S)** and **carry bit (C)**

Input		Output	
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



**The full adder:** a circuit which performs binary addition of two individual bits and an input carry bit. **carry in bit (Cin)**

A	B	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



**combinational circuits:** a circuit in which the output is dependent only on the input values

**Sequential circuit:** a circuit in which the output depends on the input values and the previous output

**SR flip-flop:**

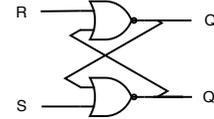
S - set R - reset

set state(self-consistent):  $Q = 1, Q' = 0$

unset state:  $Q = 0, Q' = 1$

These properties explain why the SR flip-flop can be used as a storage device for 1 bit and therefore could be used as a component in RAM because a value is stored but can be altered.

Input signals	Initial state	Final state	
S	R	Q	Q'
0	0	1	0
1	0	1	0
0	1	1	0
0	0	0	1
1	0	0	1
0	1	0	1

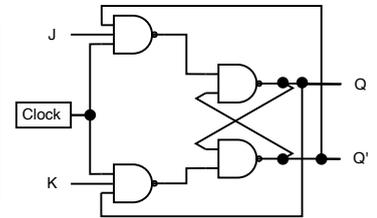


**JK flip-flop:**

**difference:** if both J and K are input as a 1 then the values for Q and Q' are toggled (they switch value).

This makes the JK flip-flop a more reliable device because there is no combination of input states that leave uncertainty as to which values are stored.

Input signals	Initial state	Final state	
J	K	Q	Q'
0	0	↑	Q unchanged
1	0	↑	1
0	1	↑	0
1	1	↑	Q toggles



**difference SR & JK:**

- SR flip-flop has an invalid combination of S and R. SR allow both Q and Q' to have the same value. SR inputs may arrive at different times.
- JK flip-flop all four combination of values for J and K are valid. JK does not allow Q and Q' to have the same value. JK incorporates a clock pulse for synchronisation

**The role of flip-flop in a computer:**

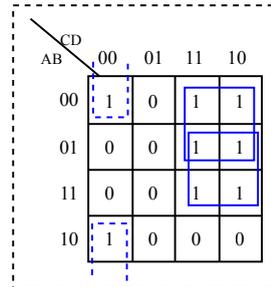
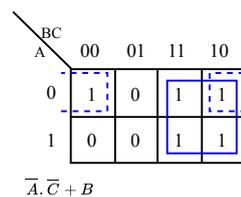
- A flip-flop can store either a 0 or a 1
- Computers use bits to store data
- Flip-flops can therefore be used to store bits (of data)
- Memory can be created from flip-flops

**Karnaugh maps(K-maps):** a method of creating a Boolean algebra expression from a truth table.

A K-map can make the process much easier than if you use sum-of-products to create minterms. If applied correctly a K-map produces the simplest possible form for the Boolean algebra expression.

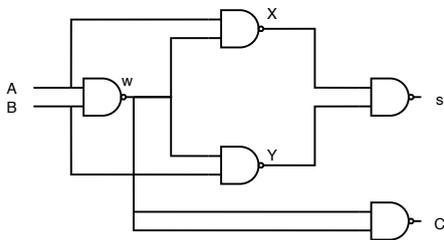
A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$\bar{A} \cdot \bar{B} \cdot \bar{C} + \bar{A} \cdot \bar{B} \cdot C + \bar{A} \cdot B \cdot C + A \cdot B \cdot C$$



**Boolean algebra:** a circuit in which the output is dependent only on the input values

Identity/Law	AND form	OR form
Identity	$1 \cdot A = A$	$0 + A = A$
Null	$0 \cdot A = 0$	$1 + A = 1$
Idempotent	$A \cdot A = A$	$A + A = A$
inverse	$A \cdot \bar{A} = 0$	$A + \bar{A} = 1$
Commutative	$A \cdot B = B \cdot A$	$A + B = B + A$
Associative	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$	$(A + B) + C = A + (B + C)$
Distributive	$A + B \cdot C = (A + B) \cdot (A + C)$	$A \cdot (B + C) = A \cdot B + A \cdot C$
Absorption	$A \cdot (A + B) = A$	$A + A \cdot B = A$
De Morgan's	$\overline{A \cdot B} = \bar{A} + \bar{B}$	$\overline{A + B} = \bar{A} \cdot \bar{B}$
Double Component	$\overline{\bar{A}} = A$	$\overline{\overline{A}} = A$



Step 1 NAND Truth Table

NAND Truth Table		
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Step 2 Truth Table -> Logic Expression

$$W = \bar{A} \cdot \bar{B} + \bar{A} \cdot B + A \cdot \bar{B}$$

Step 3 Logic Expression X and Y

$$X = \bar{A} \cdot \bar{W}$$

$$X = \bar{A} \cdot (\bar{A} \cdot \bar{B} + \bar{A} \cdot B + A \cdot \bar{B})$$

$$X = 0 + 0 + A \cdot A \cdot \bar{B}$$

$$X = A \cdot \bar{B}$$

$$X = A + \bar{B}$$

$$Y = \bar{A} + B$$

Step 4 Logic Expression S

$$S = \bar{X} \cdot \bar{Y}$$

$$S = \overline{X + Y}$$

$$S = \overline{(A + \bar{B}) + (\bar{A} + B)}$$

$$S = \bar{A} \cdot B + A \cdot \bar{B}$$